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Computer Simulation of an Electrostatic Generator: Example Case

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A test case of the LLNL Electrostatic Generator system was run on an in-house simulation code, the results of which can be used as a check on other E-S Generator simulation codes. This report describes the electrical circuit employed in the calculation, its parameters, and selected results. These results have not been checked against an independent code (such as the SPICE code) and therefor there is no implied guarantee of the validity of the data summarized here.

The electrical circuit that was analyzed in order to write the code is shown schematically in Figure 1.

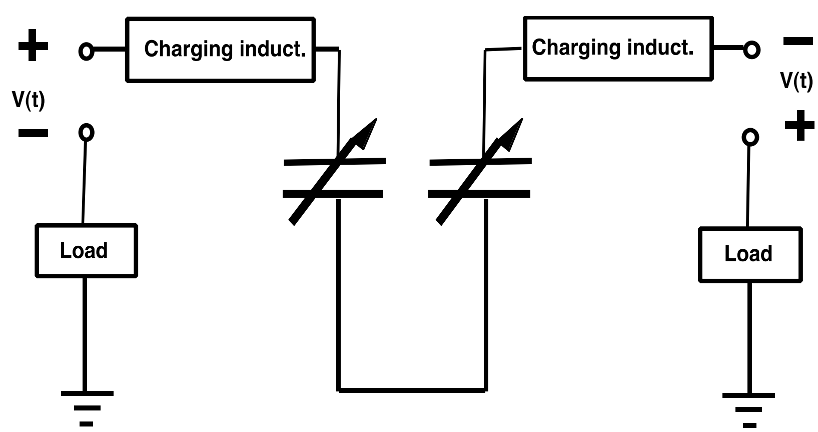

Figure 1: Schematic drawing of balanced E-S generator circuit.

The case that was analyzed was of an E-S generator suitable for wind-turbine use. In the original calculation the stator and rotor consisted of a series of disc-like assemblies of radially directed metal rods of 10 mm. diameter with the overlap of the rotor and stator rods lying between an inner radius of 1.0 meter and an outer radius of 2.0 meters. The minimum gap between a rotor rod assembly and the adjacent stator rods on either side of a rotor rod was 2.5 mm. and the mean azimuthal spacing between the rods in both the rotor and the stator assemblies was 20 mm. The length of the entire generator was 6.0 meters. It was assumed to be operated in an evacuated chamber at a shaft speed of 30 rpm.

Looking at the schematic in Figure 1 we identify the lower electrode of each condenser shown as the left and right faces of the rotor rods, while the upper electrodes correspond to the stator rod surfaces that face each rod assembly. In the original code the capacities were calculated using an approximation based on an analytical solution of rod-rod capacity. However for the results reported here the capacity function that was used in the code was an analytic fit to the original code-computed capacity. Checks between the two formulations showed that the results for generated power obtained agreed to 3 significant figure accuracy. Therefore in this report we shall give the results calculated using the analytic fit to the time-varying capacity. As a result no uncertainty as to the capacity function used in our calculations will exist for anyone in comparing their simulation results with ours.

Operating Parameters of Simulated System

As noted, the simulated rod-rod system operates at a rotational speed of 30 rpm. The azimuthal count of the rods is 471 so that the operating frequency of the system is f_0 = 235.5 Hz. The simulation program was written so that the charging voltage was set by the requirement that the maximum voltage between the rotor and stator rods (occurring at azimuthal positions located approximately halfway between the minimum-gap positions and the maximum gap positions) was limited to 100 kV. For the data reported here this meant that the charging voltages were set to a value of 62.872 kV (positive for the left side of the circuit of Figure 1, negative for the right side). To diminish the possible effects of "start-up" transients in the numerical solution ("NDSolve" in Mathematica®) of the Kirchoff Law differential/algebraic equations of the circuit, and to simplify the initial conditions on those equations that were solved by the code, the charging voltage was "switched on" at t = 0 by the function $V_0(t)$ which had the form given in Equation 1 below.

$$V[t] = V_0 \left[1 - Exp[-(\frac{t}{t_0})^2\right] \qquad (1)$$

Here $t_0 = 20.0(\pi/\omega)$, with $\omega = 2\pi f_0$. Figure 2 is a plot of this function.

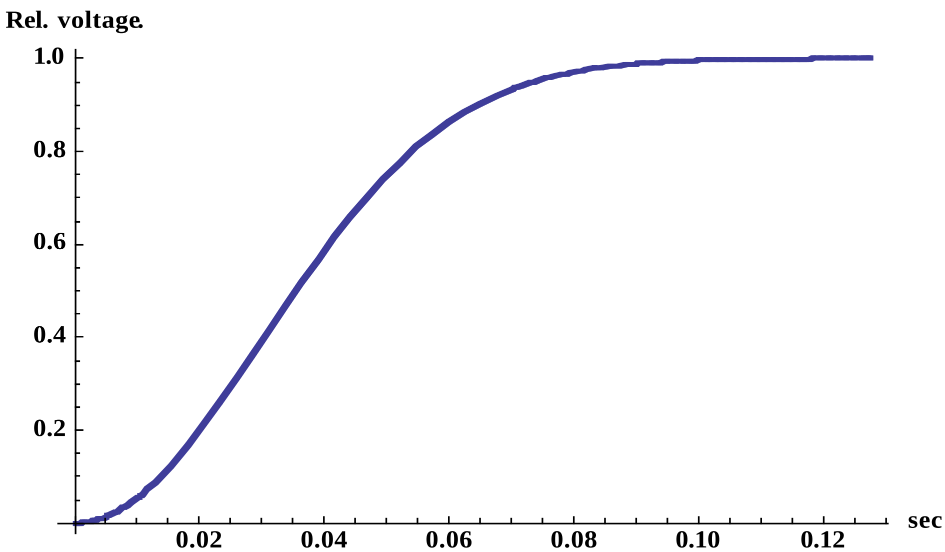

Figure 2: Normalized plot of turn-on voltage used in simulation code.

Values of the Circuit Constants

Given the charging voltages, the voltage limits, and the frequency of operation of the circuit, we may specify the values of the inductances, resistances, and capacitances of the components shown in the circuit drawing, Figure 1.

First, for the capacitances the maximum value of each capacitor, as calculated from the approximate analytical rod-rod capacity formula used, was 19.4976 mfd. The minimum value, also calculated using the same formula, was 17.9304 mfd. The ratio between the maximum and minimum value then is 1.08741, and the arithmetic average value is 18.714 mfd. As noted above these values were used to define the constants in an analytic form that was then used in the code in calculating the currents and voltages in the circuit in steady state. This analytic capacity function is given below in Equation 2.

$$C[t] = C_{max}\{1 + k * Cos[\omega t]\}/(1 + k) \qquad (2)$$

The value of the constant, k, as calculated from the maximum and minimum values of C[t], is 0.0418749. A plot of the function, Equation 2, is shown below in Figure 3.

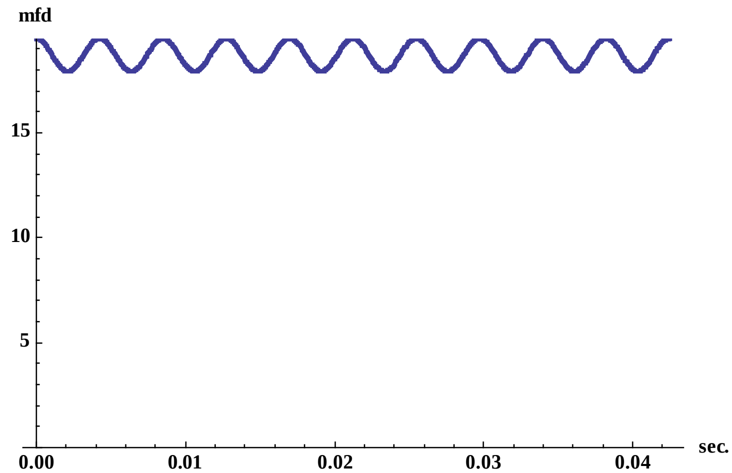

Figure 3: Plot of analytic fit to capacity variation with time.

The value of the inductances on the left and right side of the circuit of Figure 1 was determined by calculating the resonant frequency of the series-resonant circuit comprised of these inductances in series with the two time-varying capacities (evaluated when these capacities have their average value), and then adjusting the inductances so that the resonant frequency of this series circuit is equal to the operating frequency, 235.5 Hz. The series inductors thus each had a calculated value of 24.4057 millihenrys. The Q value of these inductances was taken to be 400 at the operating frequency so that the equivalent series resistance of each inductance is 0.0902823 ohms.

The left and right loads were chosen to represent, approximately, the effect of interposing a transformer between the E-S generator and the actual load. Each load was therefore made up of a 2.4 ohm resistor in series with an inductance the value of which was such that its reactance at the operating frequency corresponded to 5 percent of that resistance, i. e., its inductance was 81.0981 microhenrys. The choice of 2.4 ohms for the load resistance was based on the results of scanning a series of values for this resistance to find the value that resulted in the maximum power output, other parameters being fixed.

Key Results

For comparisons between the results of the in-house LLNL simulation code with other codes the key parameters are the total power output and the inter-electrode voltage at the time of capacity maximum. If these parameters, as calculated by another code, agree, then one can be assured that other results, such as efficiencies and currents and voltages throughout the circuit will also agree.

First, as to the power output and its evaluation: In our in-house code this quantity was calculated by performing a numerical integration of the square of the time-varying current in the load resistors after a steady state has been reached in the calculation (in this case over a time interval of 10 π/ω at a time equal to tmax = 1000 π/ω, dividing the result of the integration by the time interval (to find the average-square value of the current), and then multiplying this result by the load resistance of 2.4 ohms. This calculated power output is then multiplied by a factor of two to obtain the total power output from the generator balanced circuit.

Second, the electrode-electrode voltage at the time of minimum rotor-stator rod-rod gap (maximum capacity) can be found directly from the solution. The code-calculated value for this case is 58 kV.

The maximum electrode voltage has already been set in the code at 100 kV, occurring at a capacity value that is approximately halfway between the maximum and minimum capacity values. Figure 4 below is a plot of several cycles of the calculated inter-electrode voltage of one of the condensers as a function of time after a steady state has been reached.

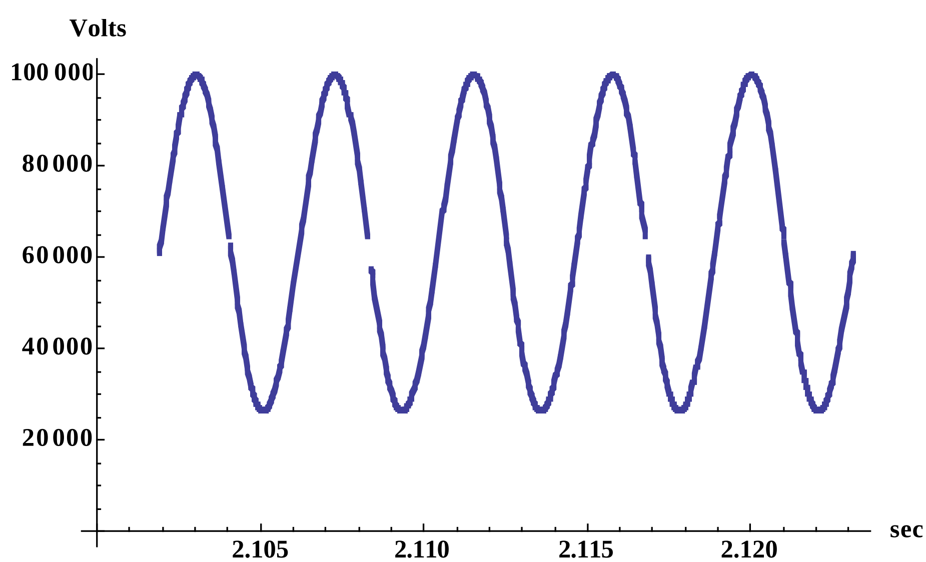

Figure 4: Plot of voltage across left condenser after steady state reached.

Third, the total output power delivered to the 2.4 ohm load resistor for this case is 2.60952 Megawatts.

Fourth, if the code is operating correctly the average dc value of the currents flowing in the circuits should tend to zero in steady state as it must from first principles. In our in-house code calculations, when these currents were evaluated they were of the order of 10^{-8} amperes, as compared to computed ac currents of order 1000 amperes. The low level of dc currents found, as compared to the ac currents, attests to the accuracy of the numerical differential equation solver and the numerical integration sub-programs in Mathematica®.

Summary

This report documents a test case, computed on an in-house computer simulation code, of one of our balanced Electrostatic Generator circuits. The report is intended to be used to make comparisons of our simulations with those made by codes developed, e.g., by groups wishing to license our technology. The circuit analyzed and the parameter values used in the simulation are given, as are selected final computed quantities, such as the power generated and the charging voltage and the electrode-electrode voltages. Agreement with our results would provide a measure of assurance of the accuracy of the other code. However, since the code results presented in this report have not as yet been subjected to an independent check employing, for example, the SPICE code, we do not guarantee their accuracy at this time.